The Minkowski sum

(applied to 2d geometry)

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Formal definition

- **A** and **B** are two sets
- **A⊕B** is the Minkowski sum of **A** and **B**

$$A \oplus B = \{a+b \mid a \in A,\ b \in B\}$$

What if A and B are 2D shapes ?

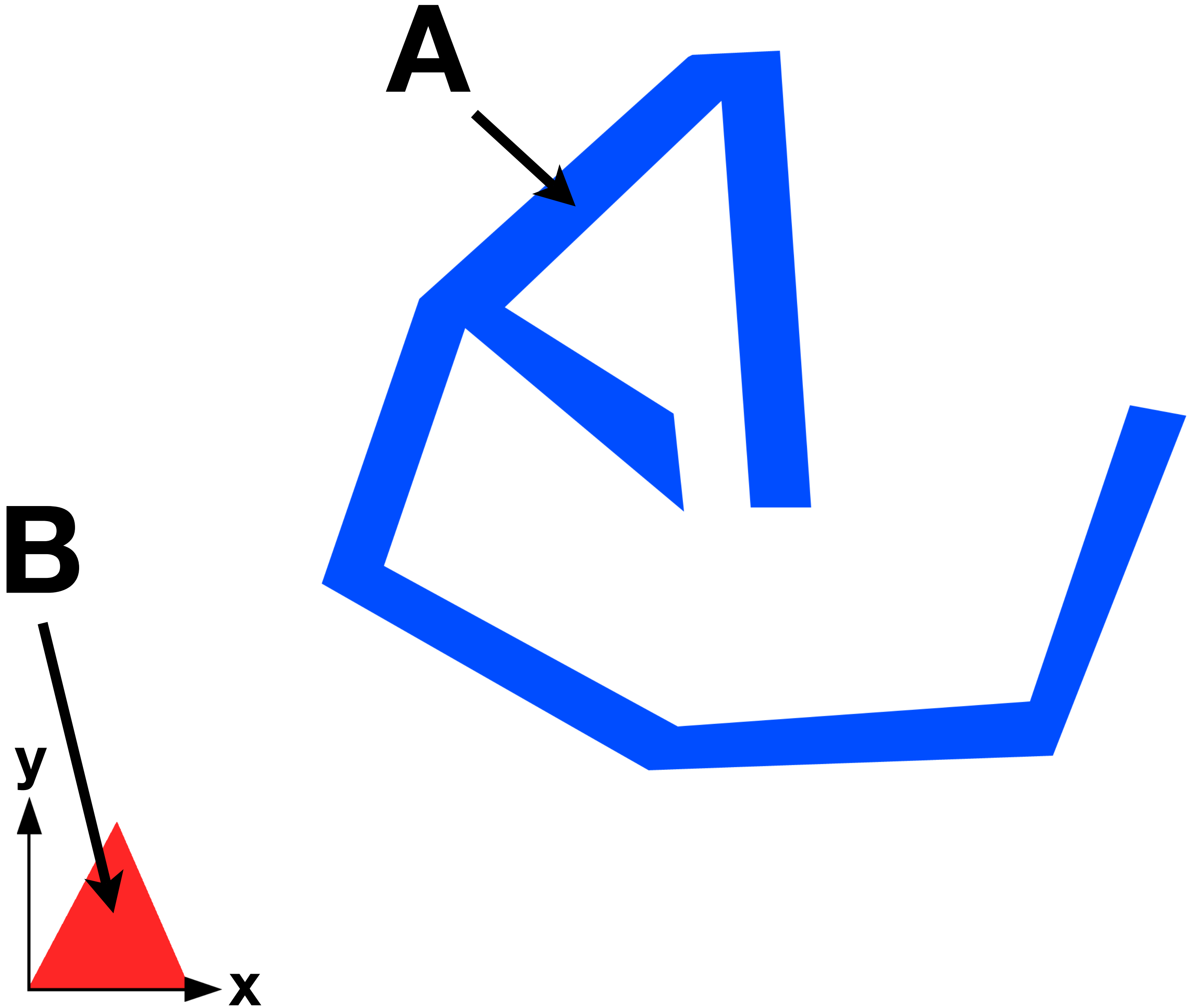
Hard to visualize ?

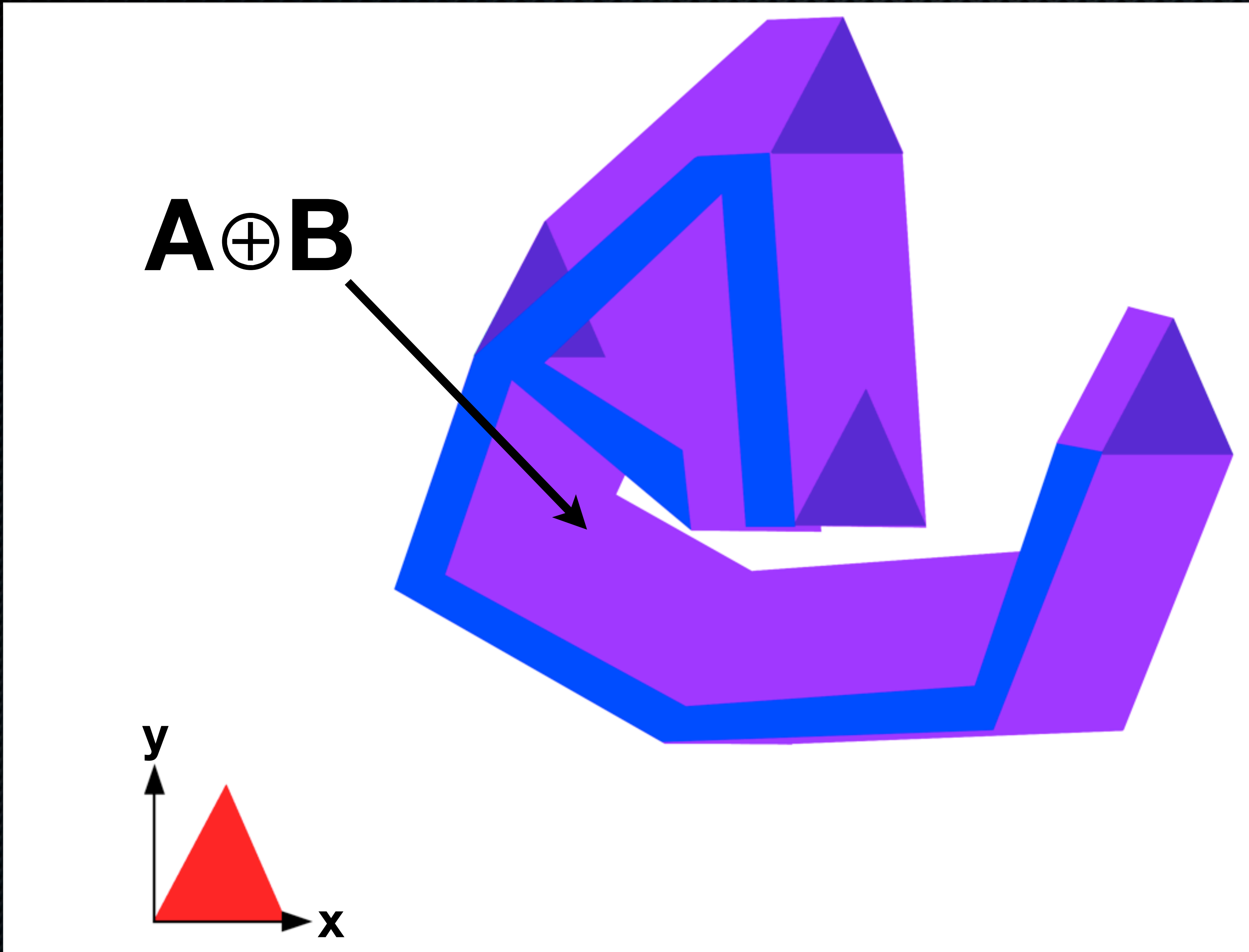
Let's see some examples...

Example 1

A is any polygon

B is a convex polygon


A
B
y
x

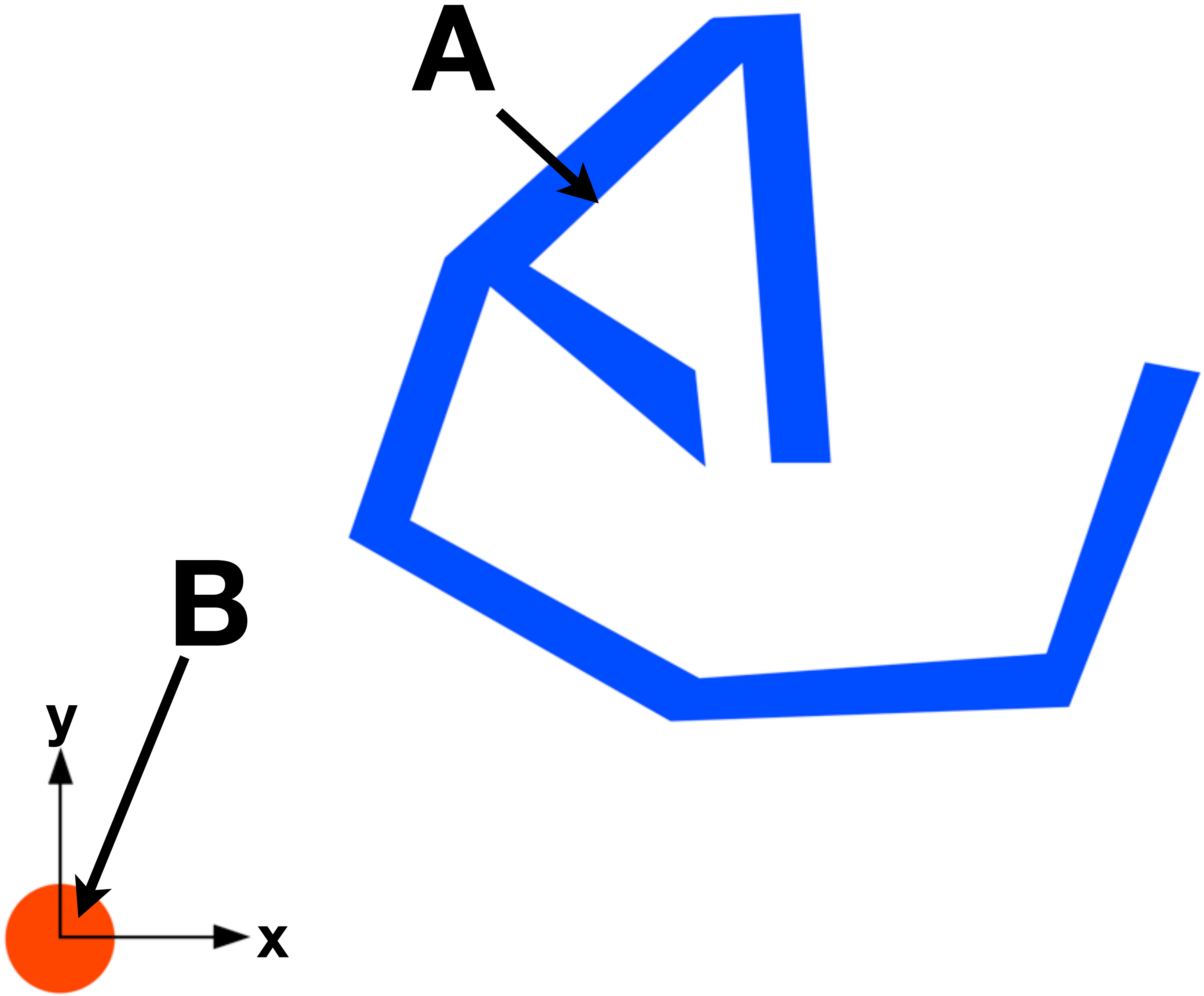


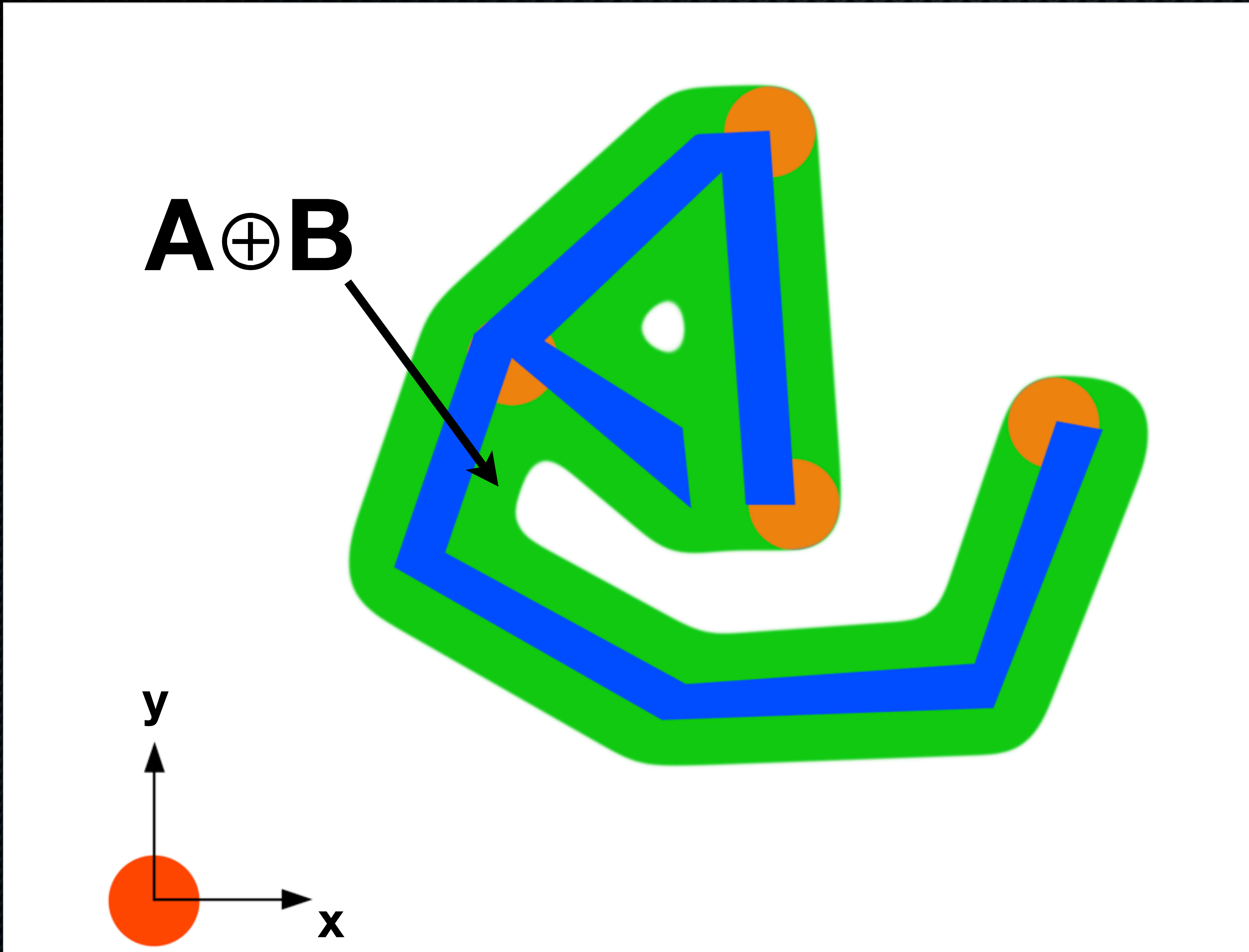
A⊕B
y
x

Example 2

A is any polygon

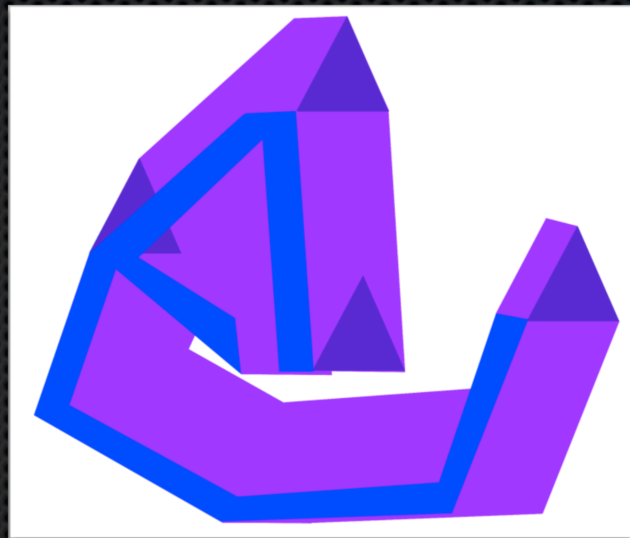
B is any disc

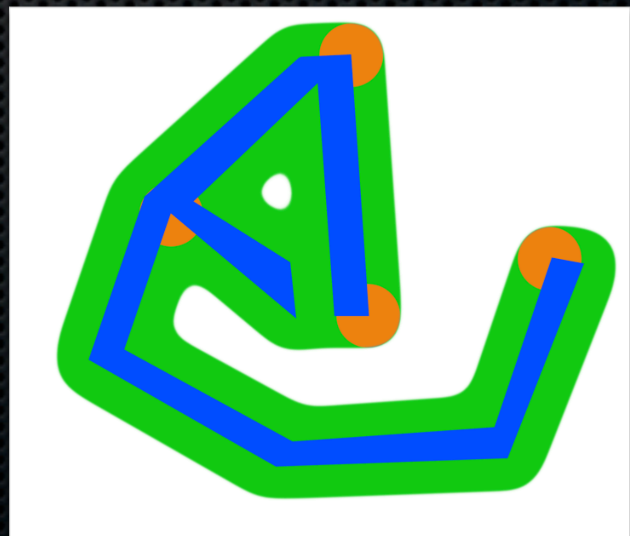

A
B
y
x


A⊕B
y
x

Intuitive definition

- What is **A⊕B** ?
- Take **B**
- Dip it into some paint
- Put its (0,0) on **A** border
- Translate it along the **A** perimeter
- The painted area is **A⊕B**





What can you do with that ?

Notably, motion planning

Free space

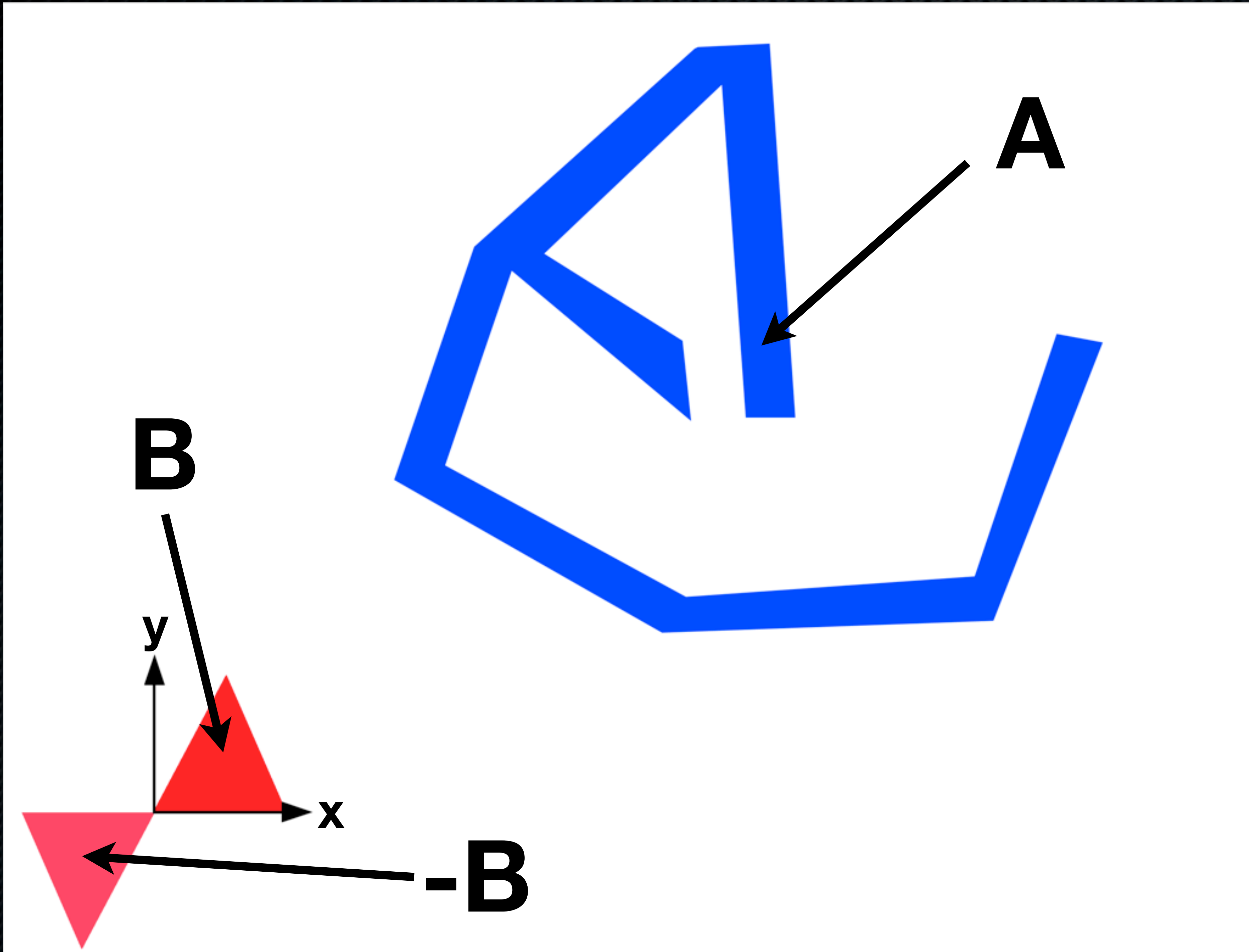
- **A** is an obstacle
  - any 2D polygon
- **B** is a moving object
  - 2D translation : **t**
  - shape : a convex polygon or a disc

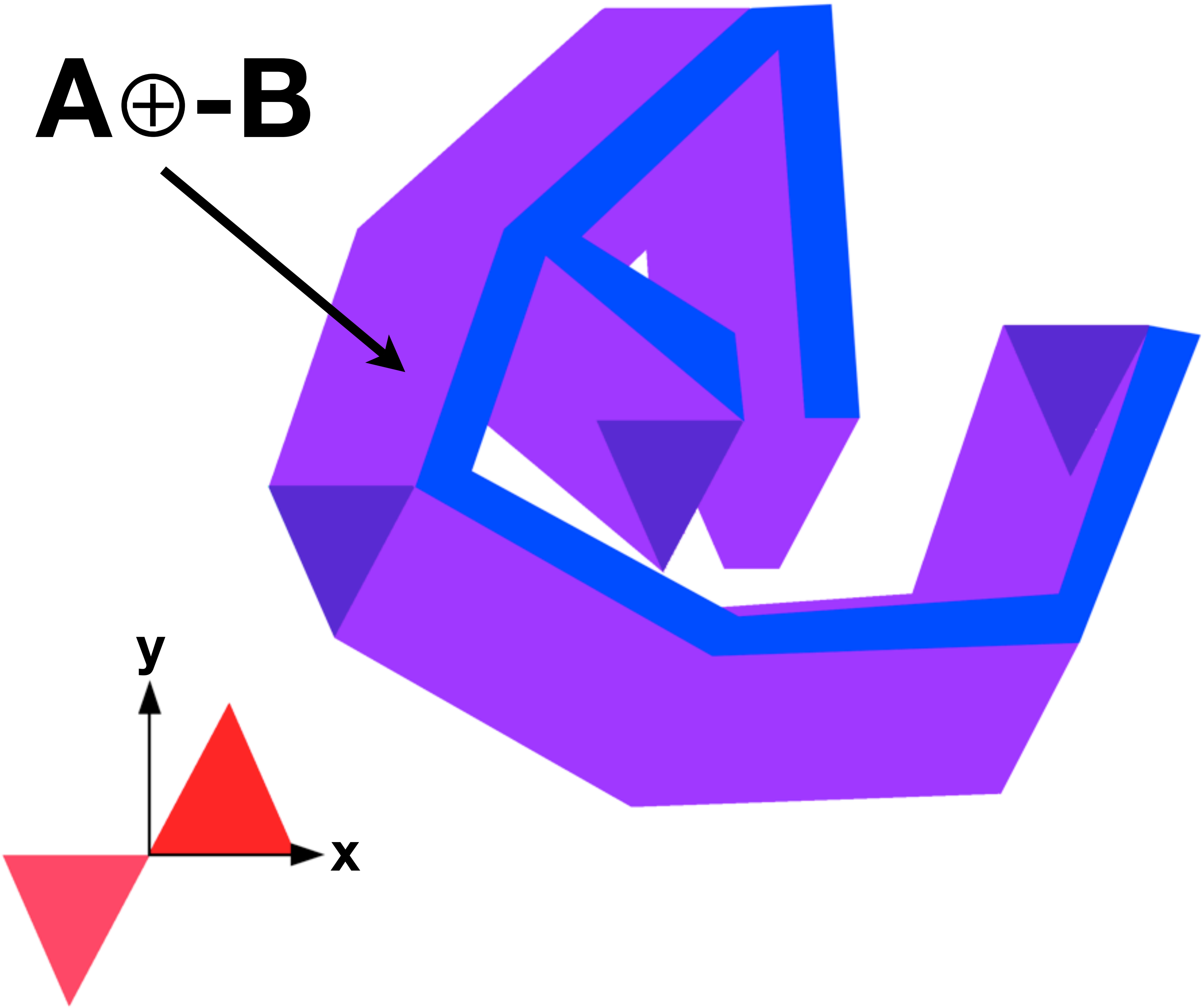
$$t \in A \oplus -B \Rightarrow \text{collision}$$

Example 1

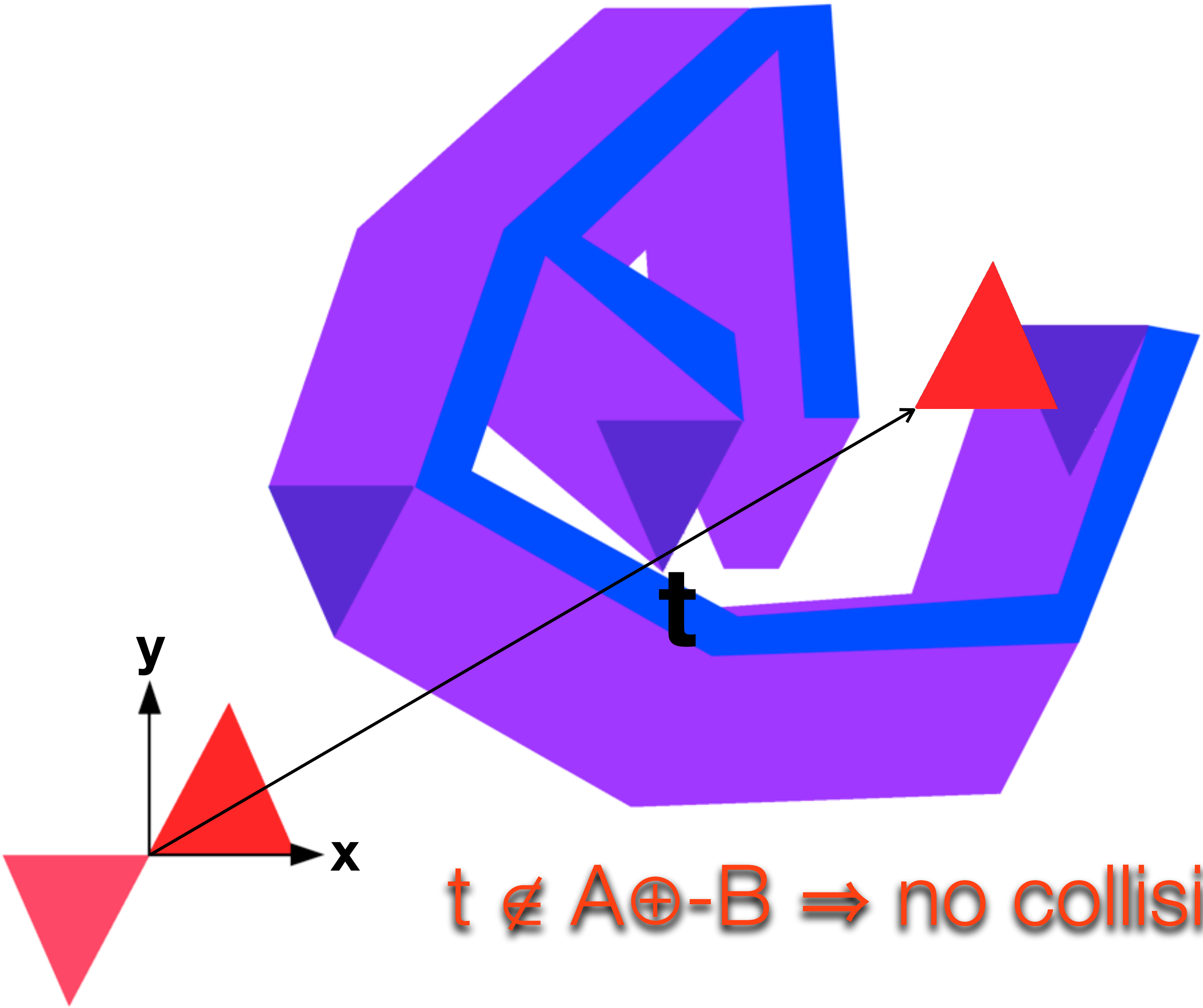
A is any polygon

B is a convex polygon

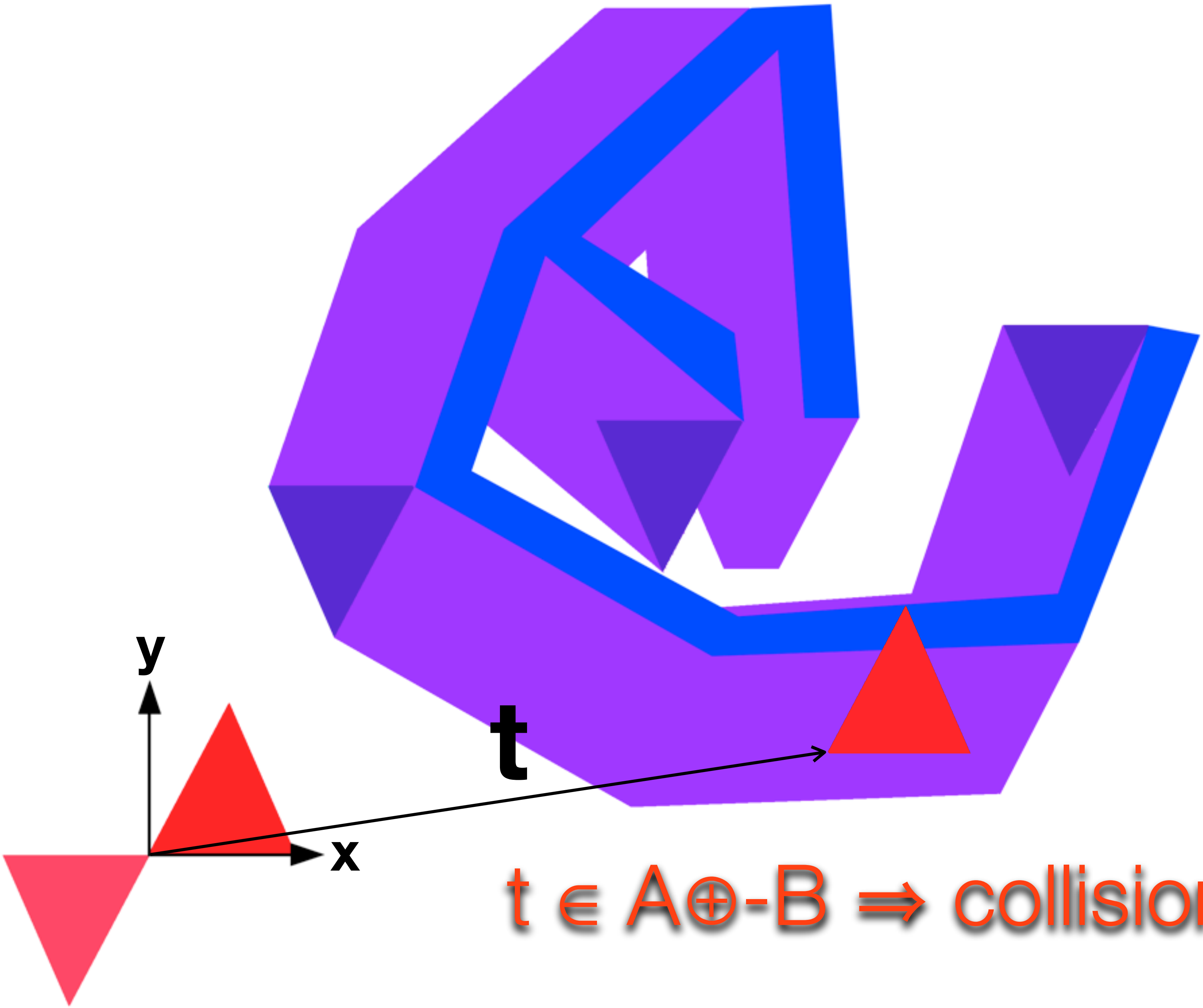

A
B
y
x
-B



A⊕-B
y
x



$t \notin A \oplus -B \Rightarrow$ no collision

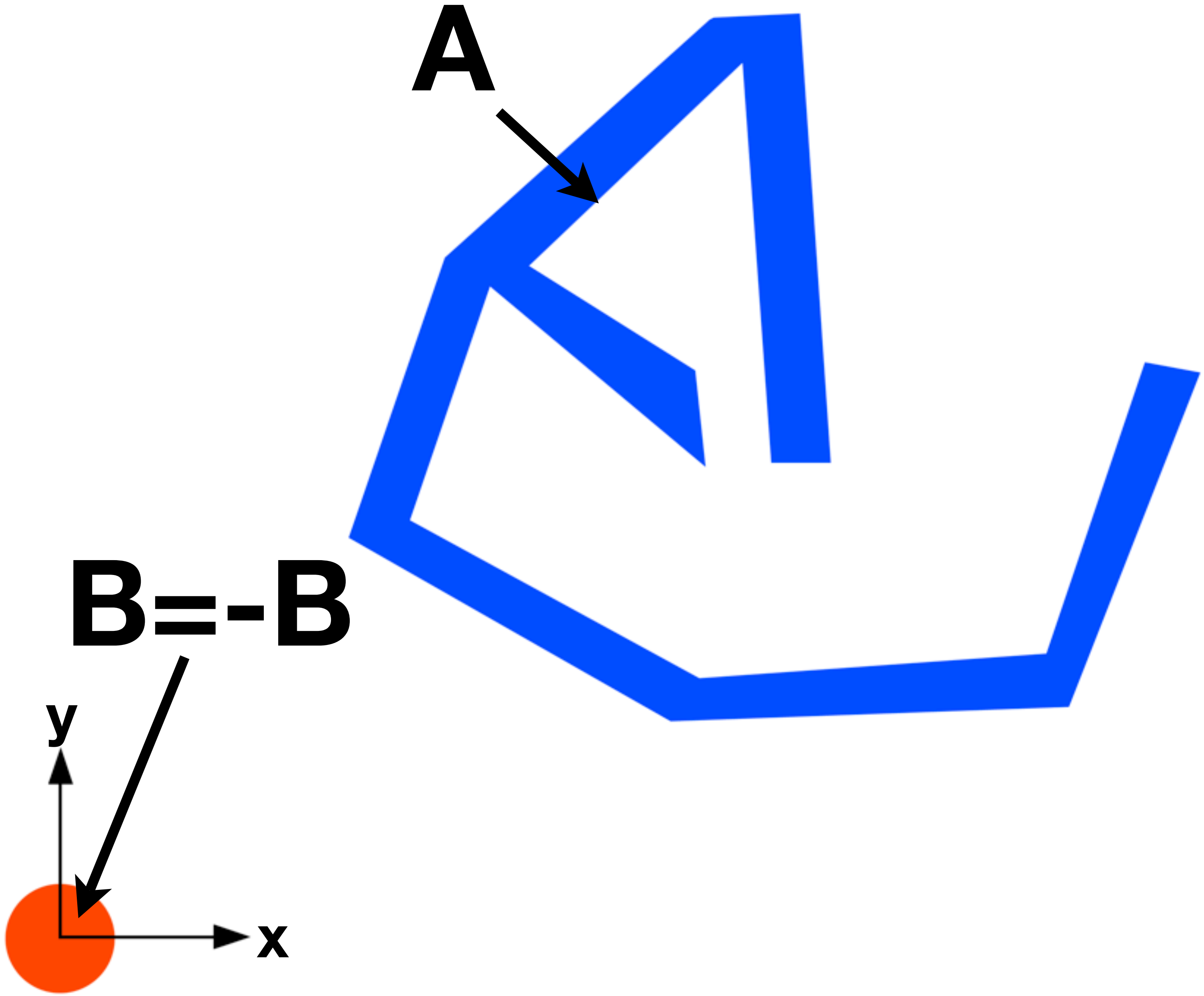


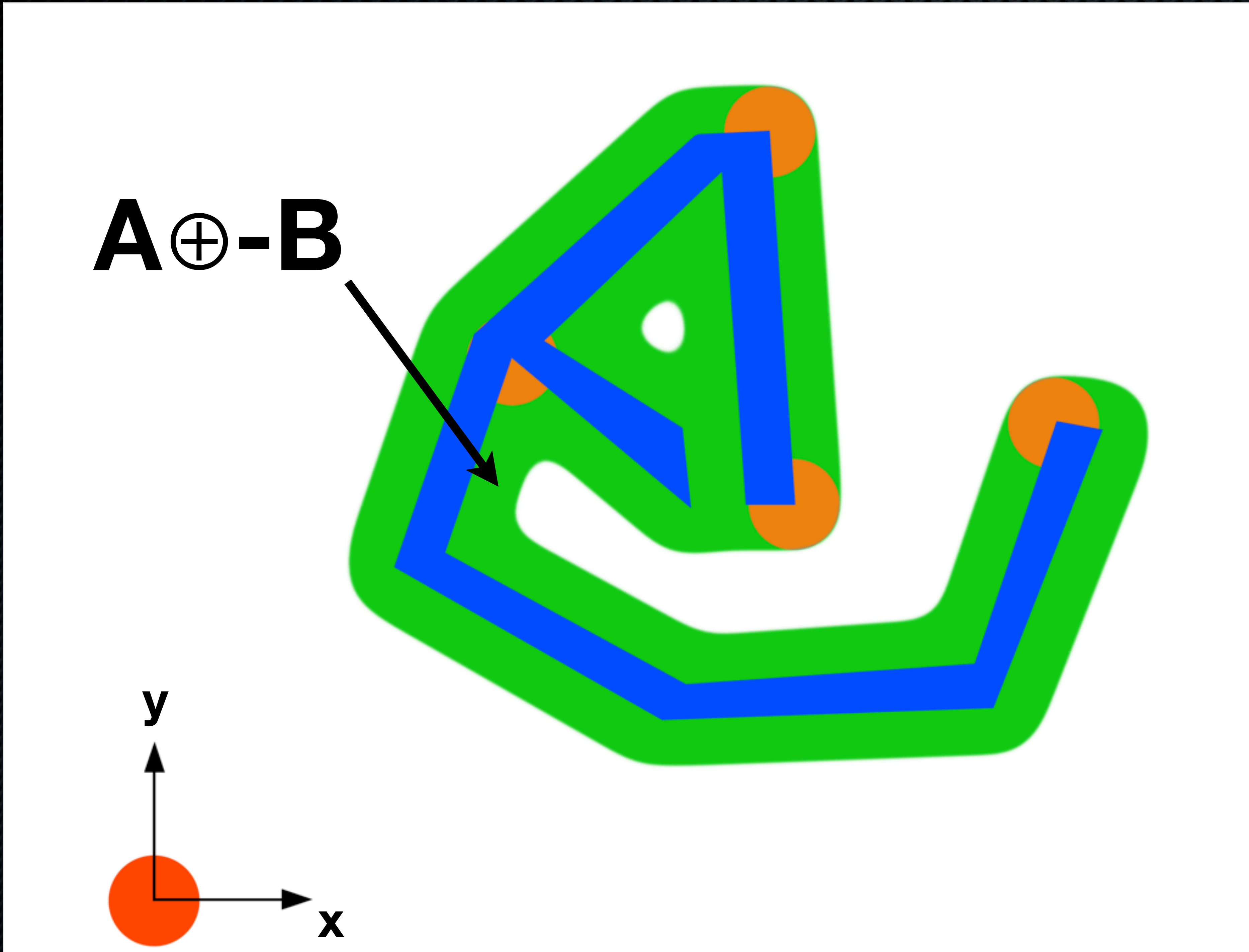
$t \in A \oplus -B \Rightarrow$ collision

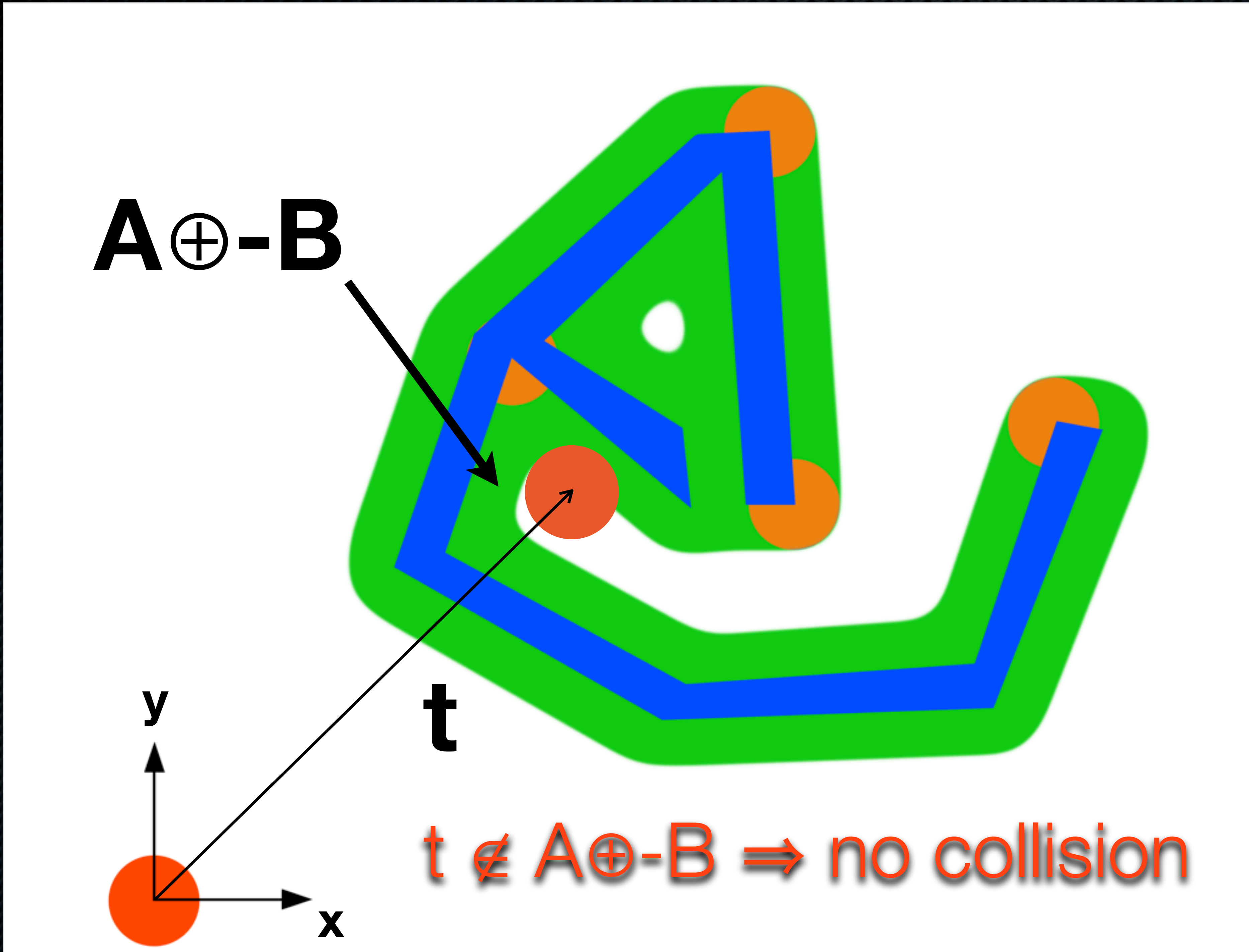
Example 2

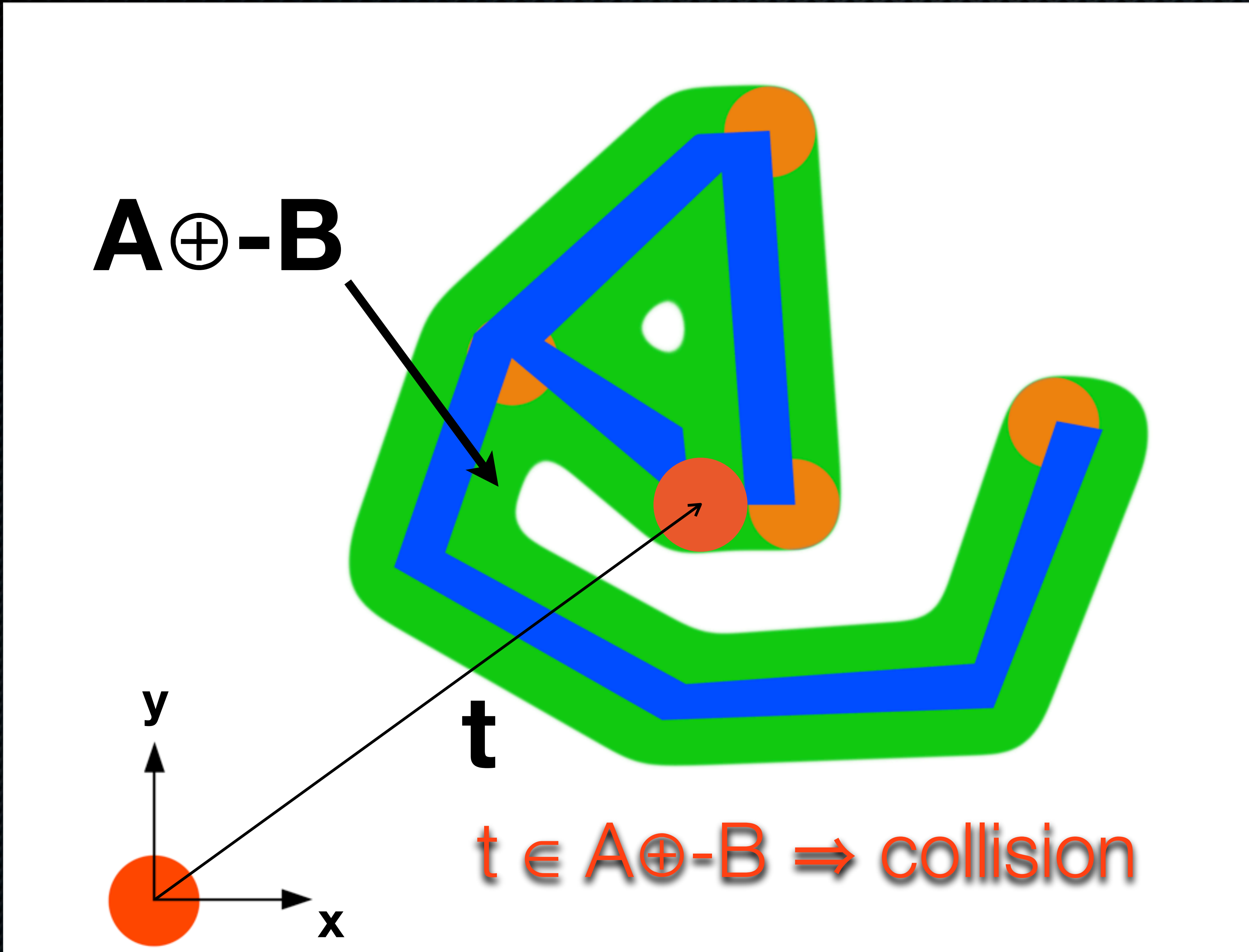
A is any polygon

B is any disc


A
B=-B
y
x


A⊕-B
y
x


A⊕-B
t
y
x
t ∉ A⊕-B ⇒ no collision


A⊕-B
t
y
x
t ∈ A⊕-B ⇒ collision

How is it computed ?

Two convex polygons

```
ConvexPolygon minkowskiSum(ConvexPolygon a, ConvexPolygon b)
{
  Vertex[] computedVertices;
  foreach(Vertex vA in a)
  {
    foreach(Vertex vB in b)
    {
      computedVertices.push_back(vA+vB);
    }
  }
  return convexHull(computedVertices);
}
```

Any polygons

- **Method 1 :** decomposition
  - decompose in convex polygons
  - compute the sum of each couple
  - the final sum is the union of each sub-sum
- **Method 2 :** convolution
  - cf. sources

Polygon offsetting

- **P** is a polygon
- **D** is a disc of radius **r**
- Computing **P**$\oplus$**D** = Offsetting **P** by a radius **r**
- Computation
  - Easy for a convex polygon
  - cf. sources

Sources

- http://www.cgal.org/Manual/3.4/doc_html/cgal_manual/Minkowski_sum_2/Chapter_main.html
- http://wapedia.mobi/en/Minkowski_addition